the time $v_1 = R^{1/2}$. For the case $\nu = 1000$ the graph of the asymptotic representation (2.5) is shown by dashes for $v_1 < R^{1/2}$, and (2.7) for $v_1 > R^{1/2}$. As already mentioned, the asymptotic form (2.5) turns out to be valid even for $v_1 = R^{1/2}$.

For a further increase in the velocity v_1 and large values of the parameter ν the solution of the problem approaches the stationary solution μ_1 which, if only the beam deflections are kept in mind, does not exhibit resonance effects when the velocity passes through the value $S^{-1/2}$. Consequently, the numerical data reflecting the passage through the velocity $S^{-1/2}$ are not presented.

In conclusion, we note that the method elucidated above can be used even to study the stresses in a beam. Thus, in place of the function u in (1.2) it is sufficient to apply the expression for the bending moments under the action of an instantaneous impulse on a beam /5, 7/ when considering the bending moments. An increase in the bending moments as the parameter ν increases will occur on passing through all three critical velocities $v_1{}^*$, $R^{1/2}$, $S^{-1/2}$.

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Translated by M.D.F.

PMM U.S.S.R., Vol.51,No.3,pp.376-381,1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

SOLITARY LONGITUDINAL WAVES IN AN INHOMOGENEOUS NON-LINEARLY ELASTIC ROD*

A.M. SAMSONOV and E.V. SOKURINSKAYA

The solution of the Cauchy problem for the equation of longitudinal displacement wave propgation in an infinitely long elastic rod is considered taking the physical and geometric non-linearities of the material, the wave dispersion, and inhomogeneities of the second and third order elastic moduli into account. A slow change in the elastic moduli in the wave propagation direction results in a perturbation of the equation of the problem solvable by the method of multiscale decomposition. It is shown that for certain initial data the solution of the problem is a soliton in the longitudinal displacement velocity. The soliton parameters are determined by the elastic moduli of the material, and its propagation over the rod is accompanied by a low-amplitude long-wave (plateau). Relations are derived between the elastic moduli for which the soliton amplitude remains constant or the plateau is not formed behind the main impulse. Under other initial conditions the Cauchy problem is solved numerically, and shaping of the solitary waves is investigated. Soliton

*Prikl.Matem.Mekhan.,51,3,483-488,1987

properties are detected in solutions of the solitary-wave type for the longitudinal velocity of displacement in the presence of slow and small changes in the elastic moduli of the material or the rod cross-sectional area.

Solutions of a simplified non-linear longitudinal wave-propagation equation in a homogeneous rod were investigated earlier /1-3/, as was the solution of the Cauchy problem on solitary wave propagation in a rod of variable cross-section /4, 5/.

1. Let us consider the evolution of a longitudinal displacement wave $\mathbf{u}=(u, v, w)$ given at the initial time $\tau=0$ in an infinite non-linearly elastic rod of radius R under the assumption that starting with a certain section $s=0$ the elastic moduli of the rod material can depend on the longitudinal coordinate s. We introduce a cylindrical system of Lagrange coordinates s, r, ξ and we assume that the strain field is characterized by a Cauchy-Green finite-strain tensor, there is no plastic flow, the rod material behaviour is described within the framework of the non-linear model of a Murnaghan elastic medium /6/. We also assume that the wave motions under consideration are axisymmetric and the characteristic wavelengths are significantly greater than the radius of the rod. Then the radial displacement v and the linear component of the axial deformation u_s are connected by the Love relationships

$$v=-\nu r u_s, \quad u=u(s, \tau) \tag{1.1}$$

(ν is Poisson's ratio). This enables the longitudinal motion in the system of dynamic equations of elasticity theory to be separated, and, integrating the free energy density over the cross-sectional area, enables us to obtain the one-dimensional Lagrangian

$$\begin{aligned} L = {} & \pi R^2 [{}^1/_2\rho u_\tau^2 + {}^1/_4\rho\nu^2R^2u_{\tau s}^2 - {}^1/_2Eu_s^2 - {}^1/_6\beta u_s^3 - \\ & {}^1/_8E\nu^2R^2(1+\nu)^{-1}u_{ss}^2] \\ \beta = {} & 3E + 2l(1-2\nu)^3 + 6n\nu^2 + 4m(1-2\nu)(1+\nu)^2 \end{aligned} \tag{1.2}$$

Here ρ is the density, E is Young's modulus, β is a non-linearity parameter, and l, m, n are third-order elastic moduli (Murnaghan moduli). The second component is the addition to the kinetic energy per unit length of the rod due to the inertia of the transverse motions. The fourth component describes the influence of the material non-linearity and refines the expression for the potential energy. The last component in (1.2) is the dispersion correction that takes account of the influence of transverse shear on the longitudinal motion.

We select the scale of variation of the dimensional variables such that the non-linear and dispersion corrections are of the same order of magnitude and small compared with the rod kinetic and potential energies evaluated within the framework of linear longitudinal wave theory*. (*See: Samsonov, A.M. and Sokurinskaya E.V.: Longitudinal displacement solutions in an inhomogeneous non-linearly elastic rod. Prepint 983, Fiz.-Tekh. Inst. Akad. Nauk SSSR, Leningrad, 1985). We reduce the variables of the problem to dimensionless form by the rule $\bar{f}=f/F$, where F is the scale for the dimensional quantity f and E^* is the scale for the dimensional quantity E. Then the small parameter

$$\delta^2=(R\mathrm{N}/S)^2= U\mathrm{B}/(SE^*)<1 \tag{1.3}$$

can be introduced and the Euler equation for the Lagrangian (1.2) in dimensionless variables takes the form (the bars are omitted)

$$u_{\tau\tau}=[Eu_s+{}^1/_2\,\delta^2(\beta u_s^2+\nu^2u_{\tau ss}-{}^1/_2\,E\nu^2(1+\mathrm{N}\nu)^{-1}u_{sss}]_s \tag{1.4}$$

We will estimate the influence of a weak rod inhomogeneity described by the small parameter $\varepsilon<1$: $E=E(\varepsilon s)$, $\beta=\beta(\varepsilon s)$, $\nu=\nu(\varepsilon^3 s)$, on the evolution of the solitary non-linear wave of the longitudinal displacement velocity given at the initial time. We will seek the solution of the initial-boundary value problem consisting of (1.4) and conditions of the form

$$\begin{aligned} & u(s, 0)=u_0(s), \quad u\in C^2[\mathbf{R}^1\times(0, T_1]] \\ & u_\tau(s, 0)=-24cE\beta^{-1}\alpha_0^2\,\mathrm{ch}^{-2}\,2\alpha_0 ps, \quad \alpha_0=\mathrm{const} \\ & c^2=E/\rho, \quad p^2=2(1+\nu(0))/(1+2\nu(0)) \\ & s\to\infty, \quad \frac{\partial^k u}{\partial s^k}\to 0, \quad \frac{d^k u_0}{ds^k}\to 0; \quad k=0, 1, 2, 3 \end{aligned} \tag{1.5}$$

We assume that the perturbations of the fundamental wave operator in (1.4), due to changes in the rod properties, do not exceed the dispersion and non-linear corrections proportional to δ^2, i.e., we give the relationship between the small parameters of the problem in the form $\varepsilon=o(\delta^2)$, $\varepsilon=\delta^3$ $(\delta\to 0)$. After insertion of a new unknown function $\varphi=-\beta u_\tau/(12cE)$ and new independent variables (the prime denotes the derivative with respect to εs)

$$t=2\delta^2 ps, \quad x=2p(s-c\tau)+\delta^3p^2s^2E'/E+\delta^6p^3s^3(E'/E)^2$$

this enables us to obtain the following problem from (1.4)-(1.5):

$$\varphi_t + 6\varphi\varphi_x + \varphi_{xxx} + \delta\ (\gamma x\varphi_x + \varphi\ ({}^7/_2\gamma - \varkappa)) = O\ (\delta^2) \tag{1.6}$$

$$\gamma = \gamma\ (\delta t) = {}^1/_2 E'/E, \quad \varkappa = \varkappa\ (\delta t) = \beta'/\beta$$

$$\varphi\ (x,\ t)\ |_{t=\delta^2 x} = 2\alpha_0^2\ \mathrm{ch}^{-2}\ \alpha_0 x \tag{1.7}$$

$$|\ x\ | \to \infty, \quad \varphi,\ \varphi_x,\ \varphi_{xx} \to 0 \tag{1.8}$$

Therefore, the original initial-boundary value problem (1.4)-(1.5) is reduced to a problem with a moving boundary for the perturbed Korteweg-de Vries (Kdv) equation written for the function φ proportional to the longitudinal velocity of displacement. We note that (1.6) does not describe the evolution of a localized wave (the second condition of (1.5)) in a radically inhomogeneous rod for $\varepsilon = O\ (\delta^2)$ when the KdV evolution operator will already not be fundamental; this problem requires separate consideration.

2. To construct the asymptotic solution of the problem we use an extension of the perturbation method /8/. We introduce new variables $\theta,\ T$: $\theta_x = 1,\ \theta_t = -4\eta^2\ (T),\ T = \delta t$, where η is a function to be determined and we will seek the solution of the problem in the form of a formal expansion in powers of the small parameter δ:

$$\varphi\ (\theta,\ T) = \varphi^0\ (\theta,\ T) + \delta\varphi^1\ (\theta,\ T) + O\ (\delta^2), \quad \delta \to 0$$

The solution of the Cauchy problem in the zeroth approximation is the soliton

$$\varphi^0\ (\theta,\ T) = 2\alpha^2\ \mathrm{ch}^{-2}\ \alpha\ (\theta - \theta_0) \tag{2.1}$$

$$\alpha^2\ (T) \equiv \eta^2\ (T) - \gamma \int_0^T \eta^2\ (z)\ dz, \quad \theta_0 = \theta_0\ (T)$$

where the amplitude α^2 is determined from the condition of no secular terms, i.e., the orthogonality of φ^0 and the components $O\ (\delta)$ in (1.6)

$$\alpha^2\ (T) = \alpha_0^2 \beta^{4/3}\ (T)\ E^{-2}\ (T) \tag{2.2}$$

After some reduction the equations of the higher approximations are reduced to linear inhomogeneous equations for the associated Legendre functions. The solution φ^1 has the form

$$\varphi^1 = \chi\ (1 - \mathrm{th}\ \psi) + \Phi^1\ (\psi,\ T)\ \mathrm{ch}^{-2}\ \psi \tag{2.3}$$

$$\Phi^1 \equiv -\chi\left[(1 - \psi\ \mathrm{th}\ \psi)\left(3 - \frac{1}{2\chi}\frac{d\theta_0}{dT}\right) + \psi\,(2 - \psi\ \mathrm{th}\ \psi)\right] +$$

$$(1 - \psi\ \mathrm{th}\ \psi)({}^1/_2\ \gamma\theta_0 + 4\eta^2(\eta^2 - \alpha^2)) + C_1\ (T)\ \mathrm{th}\ \psi$$

$$\psi \equiv \alpha\,(\theta - \theta_0), \quad \chi \equiv ({}^1/_6\,\varkappa - {}^1/_4\,\gamma)/\alpha$$

The asymptotic solution of the first approximation problem is found by determining $\theta_0\ (T)$ in the form (see the paper cited in the previous footnote)

$$\theta_0\ (T) = \sqrt{E\ (T)} \int_0^T \frac{2\chi + 8\eta^2\ (\eta^2 - \alpha^2)}{\sqrt{E\ (T)}}\ dT$$

The solution $\varphi = \varphi^0 + \delta\varphi^1$ damps out as $\theta \to \infty$, but has a non-zero limit behind the fundamental soliton

$$h\ (T) = \lim_{\theta \to -\infty}\ (\varphi^0 + \delta\varphi^1) = 2\chi\delta \tag{2.4}$$

i.e., a change in the elastic moduli results in the formation of an almost flat plateau with height $h \sim \delta$ behind the main impulse and a propagation velocity considerably less than the soliton velocity.

The quasistationary solution $\varphi^0 + \delta\varphi^1$ holds for $|\ \theta\ | \leqslant \delta^{-1/2}$. A solution uniformly suitable in θ is constructed by merging asymptotic expansions and consists of three parts: before the main impulse in the domain $\theta > \delta^{-1/2}$ an exponentially damped forerunner propagates, later a quasistationary solution $\varphi^0 + \delta\varphi^1$ of soliton type containing the main impulse with slowly varying parameters and the almost flat plateau, that goes over into an exponentially damped function (the "tail" of the soliton /9/) for $\theta < -\delta^{-1/2}$.

3. Let us discuss the properties of the solution found. A change in the amplitude $A\ (T) = 2\alpha^2$ of the main impulse in an inhomogeneous non-linearly elastic rod is described by the relationship (2.2) and differs from the relationship for the amplitude obtained in /10/ in that it contains a dependence of A on the non-linearity parameter β.

It is seen from (2.2) that a relationship exists between the second- and third-order moduli $\beta^2 E^{-3} = \mathrm{const}$, $\beta = \beta\ (T)$, $E = E\ (T)$, for which the amplitude A of the initial impulse and the "energy"

$$I_2 = \frac{1}{2}\int_{-\infty}^{\infty} \varphi^2\,dx$$

will remain constant while the sign of h agrees with the sign of E'. For $\beta^4E^{-3} = \text{const}$ a solution of the adiabatic approximation type /11/ of the theory of asymptotic integration of the KdV equation is obtained: the plateau behind the soliton is not formed in this case, the impulse retains its shape, but the amplitude varies slowly.

If only the geometric non-linearity due to finite strains is taken into account, then for $E' > 0$ (the material becomes stiffer) the height of the plateau is $h > 0$ and, consequently, the plateau behind the soliton corresponds to an additional compression strain wave relative to the impulse, the amplitude of the initial impulse diminishes here and the soliton loses "energy". On the other hand if the material becomes softer, then $E' < 0$ and the plateau with $h < 0$ corresponds to tension strain. The amplitude A and energy I_2 of the impulse here grow, which can result in large local stresses, bring the material into the domain of irreversible strains, and therefore, reduce the strength of the rod radically.

By restoring the sequence of the change of variables, explicit formulas can be obtained for the characteristics of the state of stress and strain of an inhomogeneous rod during solitary longitudinal displacement velocity wave propagation (see the footnote citation). We will confine ourselves to presenting estimates for the wave amplitudes of longitudinal displacement A_u, the radial displacement A_v according to (1.1), the longitudinal strain A_{u_s}, and the axial stress A_σ

$$A_u \sim \nu\delta R\beta^{-1/3}(s),\quad A_v \sim \nu\delta^2 R\beta^{1/3}(s)\,E^{-1}(s)$$
$$A_{u_s} \sim \delta^2\beta^{1/3}(s)\,E^{-1}(s),\quad A_\sigma \sim \delta^2\beta^{1/3}(s)$$

The longitudinal displacement u is found by integrating the solution $u_\tau \sim \varphi(\theta, T)$; the zone where $u(x, t) \neq 0$ is a domain expanding in time that is bounded by two functions of the form $u \sim \operatorname{th} x$ and $u \sim \operatorname{th}(x - Vt)$; we note that the velocity V exceeds the velocity of the linear waves c

$$\frac{V(s)}{c(s)} = 1 + 4\delta^2\left(\frac{\beta(s)}{\beta(0)}\right)^{4/3}\left(\frac{E(s)}{E(0)}\right)^{-2} + O(\delta^4),\quad c^2 = \frac{E}{\rho} \tag{3.1}$$

(the relative increase in the velocity $(V - c)/c$ for steel is 1-2%, but for elastic polymers (plexiglass, polystyrene) is 8-12%). Depending on the sign of E' the influence of the inhomogeneity can result in an increase or decrease in the velocity V; for $E' < 0$ and satisfaction of the condition

$$\left(\frac{E(s)}{E(0)}\right)^{1/2} + 4\delta^2\left(\frac{\beta(s)}{\beta(0)}\right)^{4/3}\left(\frac{E(s)}{E(0)}\right)^{-3/2} = 1$$

the influence of the non-linear and inhomogeneous properties of the material obviously cancel mutually, and the wave velocity is constant although the amplitude and energy characteristics of the soliton will vary.

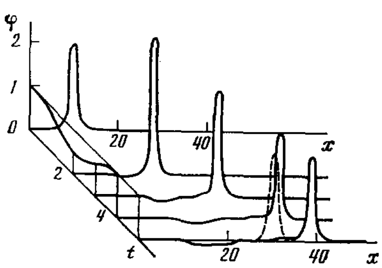

Fig.1

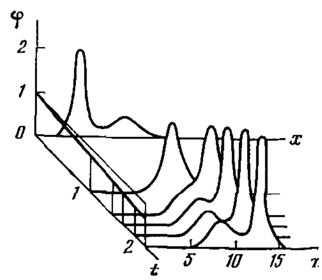

Fig.2

4. The non-stationary solutions of the Cauchy problem for (1.6) with initial conditions different from (1.7) were investigated numerically. The asymptotic and numerical solutions of the problem with soliton initial condition (1.7) were compared to estimate the accuracy of the numerical procedure. Moreover, such a comparison enables us to assess the accuracy of the asymptotic first-order solution of the KdV equation with a perturbation. A modification of the two-layer implicit scheme /12/ with order of approximation $(\Delta x^2, \Delta t)$ was used in the numerical integration. The numerical and analytic solutions were compared by confirming

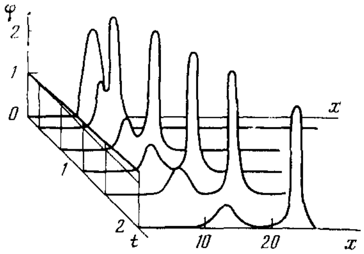

Fig.3

satisfaction of the first two conservation laws for the perturbed KdV equation and calculation of the impluse amplitude and the plateau height behind it. The results of numerical integration of the Cauchy problem on solitary wave propagation in a rod of variable section (the analytic solution is obtained in /4, 5/) and in an inhomogeneous rod showed satisfactory agreement between the asymptotic solution $\varphi = \varphi^0 + \delta\varphi^1$ and the numerical solution.

Fig.1 shows the soliton evolution in a rod for which the elastic moduli of the material varied according to the law

$$E(t) = {}^1/_2 + (1 - {}^1/_2 t)^2 - {}^1/_2 (1 - {}^1/_2 t)^4, \quad 0 \leqslant t \leqslant 4$$
$$E(t) = 1, \quad t > 4; \quad \beta(t) = E(t)$$

Along the t axis we show graphs of the function $E(t)$. For $t \leqslant 2$ the function $E(t)$ decreases, which results in a growth in the characteristics A and I_2 of the numerical solution of the problem and the formation of a plateau with $h < 0$. For $2 < t \leqslant 4$ the function $E(t)$ increases up to the initial value $E(0) = 1$, the amplitude A and energy I_2 fall to the initial values, and a plateau with $h > 0$ forms behind the impulse. The dashed lines denote the soliton location that it would occupy when propagating in a homogeneous rod $(E(t) = 1)$; it is seen that in this case the soliton would traverse a smaller distance, therefore, the magnitude of the shear might be a measure of the inhomogeneity of the rod material.

Problems on the collision of two solitons in an inhomogeneous rod were solved numerically for $\beta(t) = E(t)$ (Fig.2) and in a rod of variable section. It turns out that even in the presence of perturbations the impulses remain unchanged after interaction, i.e., possess the fundamental property of the localized solutions of the unperturbed KdV equation- the soliton property.

Numerical experiments on soliton collisions in a rod showed the strong influence of inhomogeneity on the magnitude of the wave shift in space with respect to the location each would occupy in the absence of the other. If $E' > 0$ this shift increases compared with the same quantity in the homogeneous case, while the system of two interacting solitons accelerates. For $E' < 0$ the phase shift decreases while the system of impulses is retarded; therefore, a change in the phase shift because of inhomogeneity can be considered the sole "memory" of a collision under conditions of varying parameters of the rod material.

Analysis of the results obtained in a numerical integration of the KdV equation with a perturbation enables us to describe the influence of the rod inhomogeneity on the soliton formation process from an arbitrary initial impulse. The leading front of the impulse first becomes gradually more steep, then the original wave separates into several impulses with different amplitudes and velocities. These impulses diverge and acquire the characteristic shape of solitons, where the influence of the inhomogeneity can result in both magnification of the impulses and their gradual damping. In particular, for magnification the maximum of the soliton amplitudes can exceed the amplitude of the initial impulse by a factor of 2-4.

Fig.3 shows the process of formation of two solitary waves from an initial perturbation of the harmonic-function type in an inhomogeneous rod; the elastic moduli are connected by the relationship $\beta^2 E^{-3} = \mathrm{const}$, $E' < 0$. The impulses are in the order of increasing amplitude where the property of conservation of the amplitude and energy of the solution for the mentioned connection between β and E, found for a single soliton, also turns out to be satisfied, in such a system: when the formation process is completed the amplitudes of the two solitons being formed and the energy of the solution remain constant.

The authors are grateful to N.V. Zvolinskii, S.V. Nesterov and the participants in the seminars held to discuss the results of the research.

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Translated by M.D.F.

PMM U.S.S.R., Vol.51, No.3, pp.381-385, 1987
Printed in Great Britain

0021-8928/87 $10.00+0.00

ON OPTIMAL PLASTIC ANISOTROPY*

N.V. BANICHUK and V.V. KOBELEV

An approach is developed for the optimal design of a structure, based on optimization of the anisotropic plastic properties of materials. Problems of maximizing the ultimate plastic rupture load as a result of optimal orientation of the plastic anisotropy axes in the structure elements are formulated. Necessary conditions are presented for optimality in the three-dimensional problem of the theory of ultimate plastic equilibrium. Cases of the torsion and bending of plastic rods are considered. The bilateral achievable estimates of the ultimate loads are obtained. It is noted that the conditions for achieving the upper and lower bounds agree with the necessary optimality conditions. It is proved that the maximum ultimate load is realized in the case when the direction with the greatest yield point of the material agrees with the direction given by the tangential stress vector at the time of exhaustion of the carrying capacity.

1. Formulation of the problem. Optimality conditions. We consider a deformable body that occupies a domain Ω with boundary Γ. The body material is considered to be ideally elastic-plastic. The flow state occurs at a certain point if the flow condition

$$g(\sigma_{ij}, k) \leqslant 0 \tag{1.1}$$

is satisfied with the equality sign ($g = 0$). If $g < 0$ then the material behaves elastically. Here k is the plasticity constant, g is a given function, and σ_{ij} are stress tensor components The equation $g(\sigma_{ij}, k) = 0$ in the stress space yields a family of convex surfaces enclosing the origin. It is assumed in the problems studied below that flow domains occur when loads are applied to the body. The very appearance of flow domains is considered allowable, however, it is required that the plastic strains should not result in exhaustion of the carrying capacity and to body rupture. Exhaustion of the carrying capacity is understood to be unbounded growth of strains under constant loads (/1, 2/).

To estimate the carrying capacity, the theorem on ultimate equilibrium is used, according to which the body sustains applied loads if a safe statically possible field of stresses σ_{ij} exists, i.e., a stress distribution satisfying the equilibrium equations and boundary conditions

$$\sigma_{ij,j} + q_i = 0,\ (\sigma_{ij}n_j)_{\Gamma_\sigma} = T_i \tag{1.2}$$

and such that

$$g(\sigma_{ij}, k) < 0 \tag{1.3}$$

Here n_i are components of the unit external normal vector to the body surface $(n_in_i = 1)$, and Γ_σ is the part of the body surface on which the loads T_i are given. On the rest of the

Prikl.Matem.Mekhan., 51, 3, 489-495, 1987